# Poverty: Looking for the Real Elasticities

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# Poverty: Looking for the Real Elasticities

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†The document has been realized with LaTeX. All estimations are made with R. Estimation scripts are available upon request.

**Abstract**

*After decades of intensive research on the statistical size distribution of income and despite its empirical weaknesses, the lognormal distribution still enjoys an important popularity in the applied literature dedicated to poverty and inequality. In the present study, we emphasize the drawbacks of this choice for the calculation of the elasticities of poverty. Using last version of WIID database, we estimate the growth and inequality elasticities of poverty using 1,842 income distributions under fifteen rival distribution assumptions. Our results confirm that the lognormal distribution is not appropriate for the analysis of poverty. Most of the time, it implies an overestimation of the elasticities and bias our estimation of the relative impact of growth and redistribution on poverty alleviation.*

# 1 Introduction

Midway to the end of the Millennium Development Goals (MDG), it seems that the prime objective of halving extreme poverty will be achieved in 2015. Even if the estimated levels of poverty are still debated (Chen and Ravallion 2004, Bhalla 2004, Sala-i Martin 2006), all authors agree that poverty has largely declined during the last decade. Half of the way is done, but this result conceals the great heterogeneity of observed outcomes in the developing countries in terms of poverty reduction. Whereas Asia greatly contributed to the reduction of the world poverty headcount, many African countries do not registered any improvement. In half of the Sub-Saharan African countries, extreme poverty has even worsened during the 90's. So, the poverty file is not closed and there is much to do to improve our understanding of poverty and suggest the appropriate treatment.

From an analytical point of view, poverty is directly linked to mean income and inequality. As the effects of any variable on poverty are channeled through mean income and its distribution, it is important to know how they are determined, and to investigate the sensibility of poverty measures to variations of these two elements so as to choose the most efficient policies for poverty reduction. Since the international community has decided to target its interventions on poverty alleviation, many famous studies like Ravallion (2001) or Dollar and Kraay (2002) have considered growth as the most efficient way of fighting poverty. This focus on growth is perfectly illustrated by the 2005 Word Development Report (World Bank 2005) in which the investment-growth-poverty relationship is the main motto. Such a partial analysis of the poverty issue, based on the growth-poverty relationship, led some authors to the conclusions that the MDG only resulted in pure rhetorical changes for the biggest multilateral institutions since their recommendations and politics did not evolve. At the same time researchers were largely involved in the justification of growth-oriented policies in terms of poverty alleviation, but the measure of the efficiency of these policies has been greatly debated. As Dollar and Kraay (2002) argued that the income of the poor grew at the same rate as mean income, the estimation of a mean value for the growth elasticity of poverty is widely discussed. Besley and Burgess (2003) suggest the use of a value of $-0.7$ but Bhalla (2004) gets an mean elasticity of $-3.4$. The difference is not meaningless since the achievement of the objective of halving extreme poverty under the first result implies a rate of growth which is five time larger than the one corresponding to the elasticity calculated by Bhalla (2004)[1].

As a reaction to this omnipresence of growth, some recent studies, like Heltberg (2002), Bourguignon (2003) or Ravallion (2005), have emphasized the fundamental role of distribution features in the determination of the poverty rates. Their main message is that growth reduces poverty more efficiently in less inegalitarian countries. Lopez and Servèn (2006) also explain that the contribution of inequality reduction to poverty alleviation is higher in richer countries. So, redistribution policies and distributional consequences of growth should not be ignored and the diminution of inequalities should be considered like growth as an intermediate objective of poverty alleviation policies.

However, even if everyone agrees with the necessity of considering the distribution issue for poverty analysis, there is no consensus on the relative contribution of growth and inequality reduction to poverty alleviation. In the case of factors which contribute simultaneously to growth and inequality contraction, it may be of little interest to look for this relative contribution. It is obviously not the situation of all the determinants of growth. When some factors work in opposite direction — trade openness and financial development are frequently accused of contributing to development at the expense of widening inequalities — it is crucial to know more about the trade-off that are facing decision-makers. Of course, Dollar and Kraay's (2002) results suggest that the growth process is distribution-neutral, but, as emphasized by Kanbur and Lustig (2000), if the combination of different tools is distribution-neutral on average, it may not be the case for each tool considered independently. Looking for the elasticities of poverty is therefore necessary to build efficient policies to fight poverty.

A direct estimation of these elasticities for different values of mean income and different degree of inequality can easily be achieved under the assumption that the observed distributions can be described by a known statistical distribution. In most studies (Quah 2001, Bourguignon 2003, Epaulard 2003, Kalwij and Verschoor 2005, Lopez and Servèn 2006) the lognormal distribution is used. This is a curious choice since these authors choose to set aside all the XX$^{\text{th}}$ century debates on the statistical size distributions of income. Since late XIX$^{\text{th}}$ century and the pioneering works of Pareto, research has been extremely active to retrieve the functional form which fits best the observed distributions. Practical considerations and considerable influence of the study of Aitchison and Brown (1957) may still explain the current popularity of the lognormal distribution, but cannot justify its systematic use in empirical studies. Many authors have pointed out its empirical weaknesses and have suggested alternative functional forms[2] like Maddala and Singh (1976), Dagum (1977) or the general-

[1]According to Besley and Burgess (2003), halving extreme poverty requires that the developing world grows at an annual rate of 3.8% between 1990-2015. With the elasticities suggested by Collier and Dollar (2001) and Bhalla (2004), the needed growth rates is only 1.4% and 0.8% respectively.

[2]A quite comprehensive survey is Kleiber and Kotz (2003).

ized beta 2 distribution (McDonald 1984). For instance, Bandourian et al. (2002) showed that the lognormal distribution was dramatically outperformed by many alternative functional forms for a relatively large sample of developed countries, even in the set of two parameters distributions. If the lognormal distribution is such a poor approximation of income distributions, elasticities obtained through the lognormality hypothesis are questionable. Of course, this is also the case for all policy recommendations based on this assumption.

In the present paper, we intend to shed light on the consequences of the use of a potentially inadequate distributional hypothesis using alternative distributions which are supposed to fit better observed distributions. Using a data set of 1,842 income distributions for 142 developed and developing countries, we conclude that the quality of our predictions can be significantly improved with more flexible functional forms. Moreover, we find that estimated elasticities under the lognormal assumption generally overestimate "real" elasticities and bias under certain conditions the estimated trade-off between pure growth and redistribution strategies for poverty alleviation in favor of the growth objective.

The paper is organized as follow. The following section introduces the methodology used for the estimation of the desired elasticities. Data and raw results are presented in section 3. Section 4 is concerned with the criterion used for the choice of an adequate functional form for income distributions and section 5 deals with the drawbacks of the lognormal hypothesis. Section 6 concludes.

# 2 Methodology

## 2.1 Calculation of the elasticities of poverty

In the present paper, we focus on absolute poverty. Any absolute poverty measure is a nonlinear combination of a poverty line, the mean income and a set of inequality parameters which fully describes the Lorenz curve $L(p)$. Our preferred measures of poverty are the widely used Foster, Greer and Thorbecke (1984) measures $P_\theta$:

$$P_\theta = \int_0^z \left(\frac{z-y}{z}\right)^\theta f(y)dy, \tag{1}$$

where $y$ is income, $z$ the poverty line, $f$ is the income density function and $\theta$ the parameter of inequality aversion. For $\theta = \{0,1,2\}$, $P_\theta$ is respectively the headcount, poverty gap and severity of poverty index. Under the hypothesis that incomes follow a known distribution, $f$ gets a functional form and the set of inequality parameters can be reduced to a few ones. Our choice for this "analytical" approach is justified in Bresson (2006). This approach allows to estimate the required elasticities individually for each observation with few information, to separate perfectly the growth and redistribution effects, and to compute inequality elasticities of poverty that can be compared in cross-section analysis. For the present paper, we are working with the following distributions: Pareto, lognormal, gamma, Weibull, Fisk, Singh-Maddala, Dagum and beta 2[3].

To get the parameters of these distributions, we can use some method of moments. With the help of the per-capita income and the Gini index, we can easily obtain the parameters of two parameters distributions like the Pareto, the lognormal, the gamma, the Weibull or the Fisk distributions. Several reasons led us to forsake this approach. The first one is that one needs more information about inequality for more than two parameters distributions. For large datasets, this supplementary information is usually given by points of the Lorenz curve, but the resolution of the resulting systems of nonlinear equations is generally cumbersome. Second, Gini coefficients are systematically truncated in the available datasets. As we used the points of the Lorenz curve to assess the quality of the fit (*cf* sec. 4), it appeared that this truncation most of the time increases the size of errors in a significant manner[4]. To avoid these shortcomings, we choose to estimate the parameters of our different distributions uniquely from the available points of the Lorenz curve. The Lorenz curves corresponding to each distribution are presented in table 1.

**Table 1: The Lorenz curve of used classical distributions**

| Name | Lorenz curve | Scale parameter |
|---|---|---|
| Pareto | $L(p) = 1-(1-p)^{1-\frac{1}{\alpha}}$ | $y_0 = \frac{\mu(\alpha-1)}{\alpha}$ |
| Lognormal | $L(p) = \Phi\big(\Phi^{-1}(p)-\sigma\big)$ | $\bar{y} = \ln\mu - \frac{\sigma^2}{2}$ |
| Gamma | $L(p) = G\big(G^{-1}(p,c,\gamma),c,\gamma+1\big)$ | $\rho = \frac{\mu}{\gamma}$ |
| Weibull | $L(p) = G_G\left(W^{-1}(p,c,\beta),c,\beta,1+\frac{1}{\beta}\right)$ | $\rho = \frac{\mu}{\Gamma\left(1+\frac{1}{\beta}\right)}$ |
| Fisk | $L(p) = B_1\left(p,1+\frac{1}{\tau},\frac{\tau-1}{\tau}\right)$ | $\kappa = \frac{\mu}{\Gamma(1+\frac{1}{\tau})\Gamma(1-\frac{1}{\tau})}$ |
| Singh-Maddala | $L(p) = B_1\left(1-(1-p)^{\frac{1}{\lambda}},1+\frac{1}{\tau},\lambda-\frac{1}{\tau}\right)$ | $\kappa = \frac{\mu\Gamma(\lambda)}{\Gamma(1+\frac{1}{\tau})\Gamma(\lambda-\frac{1}{\tau})}$ |
| Dagum | $L(p) = B_1\left(p^{\frac{1}{\theta}},\theta+\frac{1}{\tau},1-\frac{1}{\tau}\right)$ | $\kappa = \frac{\mu\Gamma(\theta)}{\Gamma(\theta+\frac{1}{\tau})\Gamma(1-\frac{1}{\tau})}$ |
| Beta 2 | $L(p) = B_{G2}\left({B_2}^{-1}(p,c,\lambda,\theta),c,1,\lambda+1,\theta-1\right)$ | $\kappa = \frac{\mu\Gamma(\theta)\Gamma(\lambda)}{\Gamma(\theta+1)\Gamma(\lambda-1)}$ |

Note: $\Phi$ stands for the c.d.f. of the standard normal distribution, $c$ for any constant term, $G$ for the c.d.f. of the Gamma distribution, $G_G$ for the c.d.f. of the generalized gamma distribution, $W$ for the c.d.f. of the Weibull distribution, $B_1$ for the c.d.f. of the Beta distribution of the first kind, $B_2$ for the c.d.f. of the Beta distribution of the second kind, $B_{G2}$ for the c.d.f. of the generalized Beta distribution of the second kind. More details on the last distributions in Kleiber and Kotz (2003).

For the derivation of income and inequality elasticities, we follow Kakwani (1993). With

[3] We also tried to use the generalized gamma, the beta of the first kind and the generalized beta of the second kind distributions. As the estimators of the non-linear least-squares were not convergent with these functional forms, we gave up using these distributions. For a closer look at the linkages between all these distributions, see McDonald (1984).

[4] Truncations and rounding are also a matter of concern for points of the Lorenz curve, but the loss of precision is less important.

the headcount index, the growth elasticity of poverty, $\eta_\mu$ is simply:

$$\eta_{0,\mu} = -\frac{zf(z)}{P_0}, \tag{2}$$

where $\mu$ stands for mean income. For $\theta \neq 0$, the elasticity is:

$$\eta_{\theta,\mu} = -\frac{\theta(P_{\theta-1} - P_\theta)}{P_\theta}. \tag{3}$$

For the estimation of the inequality elasticity of poverty, we have to deal with the problem that income distributions can change in various ways. Under a strong hypothesis of a two-parameter distribution[5] any variation of an inequality index leads to a unique Lorenz curve. So is it for the inequality elasticities[6]. However, these estimated elasticities cannot be compared through different statistical distributions since they all imply different transformations of the Lorenz curve. For more flexible functional forms, we have to chose how the Lorenz curve should move to get a unique value of the desired elasticity corresponding to given initial conditions. Kakwani (1993) suggests the following shift of the Lorenz curve:

$$L^*(p) = L(p) - \varepsilon\big(p - L(p)\big), \tag{4}$$

where $\varepsilon$ indicate a proportional change in the Gini coefficient[7]. Such transformation of the Lorenz curve implies Lorenz dominance. So, for negative (positive) value of $\varepsilon$, the situation of the poor never worsen (improve). From equation (4), Kakwani (1993) proposed the following Gini elasticities:

$$\eta_{0,G} = (\mu - z)\frac{f(z)}{P_0}, \tag{5}$$

$$\eta_{\theta,G} = \theta + \frac{\mu - z}{z}\frac{P_{\theta-1}}{P_\theta} \quad \forall \theta \neq 0. \tag{6}$$

[5]This is a strong hypothesis since we simultaneously assume that :

- the income distribution that is considered can be described by the chosen statistical distribution;
- the income distribution changes in such a way that the final distribution can also be described by the same kind of statistical distribution.

[6]For example, the "natural" Gini elasticity of the headcount index under a strong lognormality assumption is:

$$\eta^*_{0,G} = \lambda\left(\frac{\log\left(\frac{z}{\mu}\right)}{\sigma} + \frac{\sigma}{2}\right)\left(\frac{\sigma}{2} - \frac{\log\left(\frac{z}{\mu}\right)}{\sigma}\right)\frac{G}{\sigma\sqrt{2}\varphi\left(\frac{\sigma}{\sqrt{2}}\right)},$$

were $\lambda$ and $\varphi$ represent the hazard rate and density function of the standard normal distribution.

[7]It can easily be shown that $\varepsilon$ can also be interpreted as the same proportional increase of all the standardized moments of the Lorenz curve defined by Aaberge (2000) as:

$$D_t = (t+1)\int_0^1 p^{t-1}\big(p - L(p)\big)\,dp.$$

The transformation presented in equation (4) is also of great interest because it gives the possibility to compare the inequality elasticities obtained through different distributional assumptions. The interest is not only practical, since the strict respect of a particular distribution for poverty analysis is not neutral in terms of the relative importance of growth and inequality elasticities, and so may reflect some political preferences. With a common strategy for the evolution of inequality and a weak distributional hypothesis — we just assume that the current distribution can be described by the chosen statistical distribution —, estimated elasticities do not depend on the specific distributional changes corresponding to each distribution. On the figure 1 one can observe the differences between several two-parameter distributions which are used in the present study. In each quadrant, the solid line represents the Lorenz curve corresponding to a Gini coefficient equal to 0.55. The dashed and the dotted lines corresponds to the Lorenz curves respectively obtained through the same statistical distribution and through Kakwani's transformation for a 30% decrease of the Gini index.

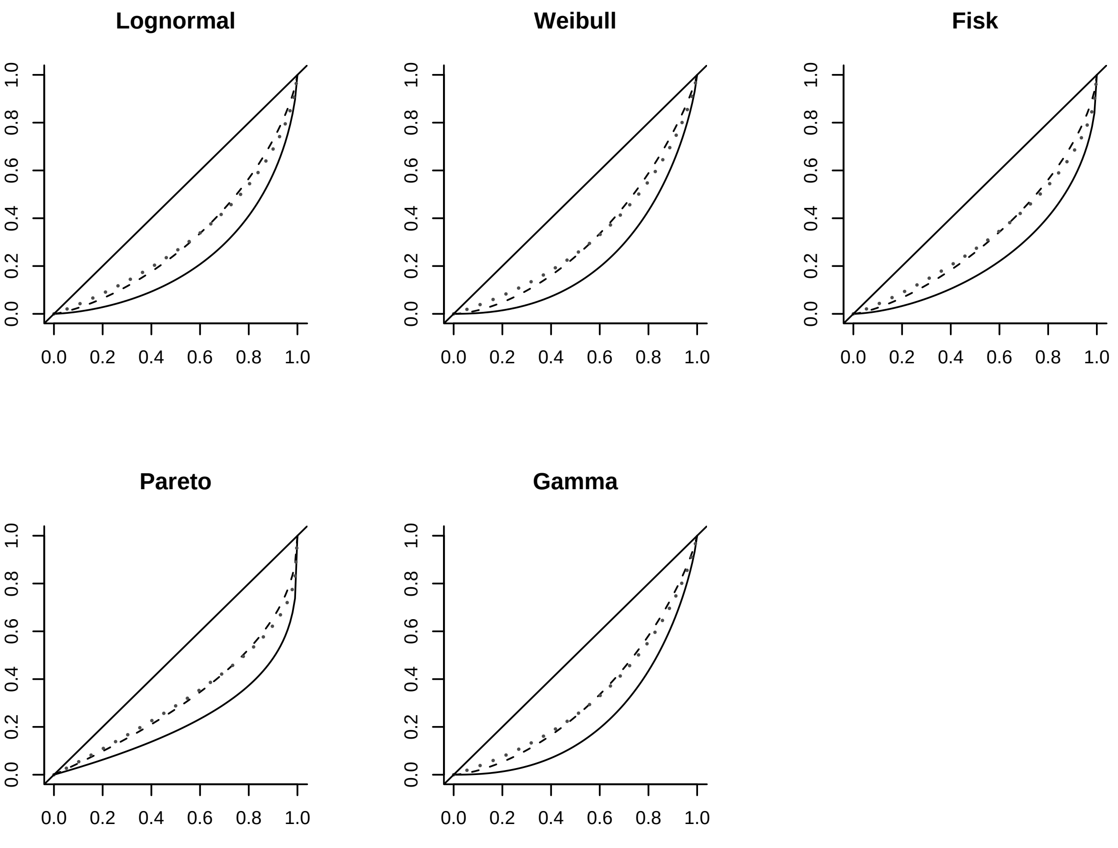



**Figure 1: "Natural" vs Kakwani's transformation of the Lorenz curve for two parameters distributions.**

In all cases, we can observe that the Lorenz curve resulting from Kakwani's transformation are more skewed toward the upper point of the Lorenz curve than the "natural" curve corresponding to the same value of the Gini coefficient. This means that the poorest benefit more from the fall of income inequality with Kakwani's transformation. Since the Foster et al. (FGT) measures are directly linked to the slope and curvature of the Lorenz curve, we should generally obtain greater inequality elasticities of poverty than those that are derived from a transformation which preserves the type of the distribution. For the three-parameter distributions that are used in the study, namely the beta of the second kind, the Maddala and Singh (1976) — also known as the Burr XII distribution— and the Dagum (1977) — also called the Burr III distribution —, elasticities are not unique. So for a same distribution, "natural" elasticities can be either larger or lower than those corresponding to the Kakwani's transformation. However, since the poor are particularly sensitive to this transformation of the Lorenz curve, we can reasonably consider that our estimated Gini elasticities will be quite high.

The other interesting feature of Kakwani's (1993) formula is that the respective importance of growth and inequality elasticities is easily predicted, especially for the headcount index. We note:

$$\frac{\eta_{0,\mu}}{\eta_{0,G}} = \frac{z}{z-\mu} \tag{7}$$

$$\frac{\eta_{\theta,\mu}}{\eta_{\theta,G}} = \frac{z(P_{\theta-1}-P_\theta)}{z(P_{\theta-1}-P_\theta)-\mu P_{\theta-1}} \quad \forall \theta \neq 0 \tag{8}$$

Equation (7) is particularly interesting since we can see that, for the headcount index, the ratio of the growth elasticity to the Gini elasticity obtained through Kakwani's transformation does not depend on the income distribution. So, it will be the same, whatever distributional assumption is made. As it only depends on per capita income, we can already know that growth policies[8] will be more efficient in terms of poverty reduction that redistributive policies when the ratio of the mean income to the poverty line is low. On the contrary, redistribution is the only effective tool for rich countries.

When $\theta \neq 0$, distribution matters. However, we can notice that the ratio is always negative. It can be easily shown that its absolute value decreases with mean income. So redistributive policies becomes more and more attractive as per capita income increases.

### 2.2 Some alternative functional forms for the Lorenz curve

In addition to known distributions, we tried to use some *ad hoc* functional forms for the Lorenz curve. Characterizing a distribution through the direct estimation of the Lorenz curve has been first used by Kakwani and Podder (1973) and has known further important developments. Most of the time, these functional forms are used for descriptive purposes, but Datt and Ravallion (1992) suggested that they could be used to estimate elasticities of poverty. These Lorenz curves can be seen as *ad-hoc* since they are generally not theoretical

[8]By growth policies, we mean policies that would lead to an increase of mean income with no distributional change. Of course, this is a pure theoretical view since observed growth always implies some redistribution.

grounded — the only exception may be Maddala and Singh (1977). However, they generally fit pretty well the data and their estimation is rather easy. Nevertheless, the use of *ad hoc* Lorenz curves raises some problems. First, the underlying distribution function may not be defined for the value of the poverty line[9]. Second, the corresponding cumulative distribution functions (c.d.f.) have sometimes no closed form. To calculate the value of the FGT measures and their elasticities, we have to use the following properties of the Lorenz curve[10]:

$$\left.\frac{\partial L(p)}{\partial p}\right|_{p=P_0} = \frac{z}{\mu}, \tag{9}$$

$$\left.\frac{\partial^2 L(p)}{\partial p^2}\right|_{p=P_0} = \frac{1}{\mu f(z)}. \tag{10}$$

Despite the attractiveness of these functional forms, Datt and Ravallion (1992) is the only study in which *ad hoc* Lorenz curve are used. In the present paper, we include the functional forms[11] described by Kakwani and Podder (1973), Maddala and Singh (1977), Gaffney, Koo, Obst and Rasche (1980), Kakwani (1980), Arnold and Villaseñor (1989), Fernandez, Garcia, Ladoux, Martin and Ortega (1991) and Chotikapanich (1993).

All the distributions and Lorenz curves used in the present paper are described in table 2. Equations for the computation of the poverty headcount for all the used distributions are presented in table 3.

**Table 2: The different *ad hoc* functional forms used**

| Name | Lorenz curve |
|---|---|
| Kakwani and Podder (1973) | $L(p) = p^r e^{-s(1-p)}$ |
| Chotikapanich (1993) | $L(p) = \frac{e^{kp}-1}{e^k-1}$ |
| Gaffney et al. (1980) | $L(p) = \left(1-(1-p)^{\phi}\right)^{\frac{1}{\zeta}}$ |
| Fernandez et al. (1991) | $L(p) = p^{\vartheta}\left(1-(1-p)^{\phi}\right)$ |
| Maddala and Singh (1977) | $L(p) = -bdp + (1-b+bd)p^a + b\left(1-(1-p)^d\right)$ |
| Kakwani (1980) | $L(p) = p - \xi p^{\nu}(1-p)^{\upsilon}$ |
| Arnold and Villaseñor (1989) | $L(p) = \frac{f\left(p^2-L(p)\right)+gL(p)(p-1)+q\left(p-L(p)\right)}{1-L(p)}$ |

Note: $\Phi$ stands for the c.d.f. of the standard normal distribution, $G$ for the c.d.f. of the Gamma distribution, $G_G$ for the c.d.f. of the generalized gamma distribution, $W$ for the c.d.f. of the Weibull distribution, $B_1$ for the c.d.f. of the Beta distribution of the first kind, $B_2$ for the c.d.f. of the Beta distribution of the second kind, $B_{G2}$ for the c.d.f. of the generalized Beta distribution of the second kind. More details on the last distributions in Kleiber and Kotz (2003).

[9] This is also a well-known feature of the Pareto distribution.

[10] More details on the use of *ad hoc* Lorenz curves for poverty analysis in Datt (1998)

[11] We also tried to estimate the parameters of Castillo et al. (1999) class of Lorenz curve, but estimators were not convergent.

# 3 Data and results

Income distribution data are from the UNU-WIDER World Income Inequality Database (version 2.0a., June 2005). Dropping observations when quality and reference population were not satisfying[12], we get a sample of 1,842 distributions for 142 developed and developing countries from 1950 to 2002. For each distribution we can make use of 6 to 13 points[13] of the Lorenz curve to estimate the parameters of the different Lorenz curves. Most functional forms imply non-linear least squares estimations, but estimators are convergent. To compute our poverty measures and scale parameters, we use PPP per capita income from Penn World Table 6.1. The main characteristics of the database are summarized in tables 11 and 12. For the present exercise, we exclusively work with the traditional 2$PPP poverty line[14, 15].

Mean elasticities for the various poverty measures and distributions are reported on table 4. Whatever poverty measure we choose, we can observe great differences between most distributions. In the case of the beta 2 distribution, the average growth elasticity of the headcount index is approximately $-5.5$ but $-1.27$ with the Weibull distribution. Bootstraped[16] standard errors shows that differences are often significant at the 5% level.

However, one should be careful in interpreting these estimated elasticities. In table 4, we can observe some erroneous values. In particular, we obtain some positive values for the growth elasticities although these are theoretically always negative. These irregular values are due to parameters which do not satisfy the validity conditions of the Lorenz curve (*i.e.* $L(0) = 0$, $L(1) = 1$ and $\frac{\partial^2 L(p)}{\partial p^2} \leqslant 0$) or intervals of definitions which do not include the poverty line. The percentage of valid estimations are reported in table 4. It appears that the Pareto distribution and the *ad hoc* functional forms for the Lorenz curve cannot be employed each one individually to analyze poverty for the whole sample. As the Pareto distributions and Kakwani and Podder (1973), Kakwani (1980), Arnold and Villaseñor (1989) and Chotikapanich (1993) Lorenz curves can only be used on a small part of the sample, we temporally exclude them from the set of tools used for the estimation of the elasticities. With the ramaining functional forms, we can concentrate on a subsample which includes 82.5% of initial observations. For this subsample, the mean income is slightly lower but the difference is not significatively different. The average value of the Gini index is quite equal.

[12] In particular, we removed many observations related to urban or rural populations.

[13] We add the (0,0) and (1,1) points since some functional forms for the Lorenz curve do not necessarily respect the conditions $L(0) = 0$ and $L(1) = 1$. In our sample, the average number of observations is 10.

[14] For international comparisons, the 1$PPP poverty line is also widely used. We prefer using the 2$PPP line because it increases the ratio of the poverty line to mean income. Since our sample includes high income countries, it seems more reasonable to adopt the most meaningful line. We also have to mention that a lower poverty line would increase the number of invalid estimations for the *ad hoc* Lorenz curves since most of them are not defined $\forall z \in \mathbb{R}^+$. A last reason is that the behavior of most distributions greatly varies at the tails. Our results would presumably be even more heterogeneous with the 1$PPP poverty line.

[15] Strictly speaking the exact value is 2.16$ in 1996 PPPs. The poverty line defined for the Millennium Development Goals is fixed for 1993 PPPs, but Penn World Tables 6.1 are based on 1996 values.

[16] For the present study, we use a two stage bootstrap procedure. In the first stage, individual elasticities are estimated on many samples with replacement of the points of the Lorenz curve. Then, the required statistics are computed on samples with replacement of the available distributions.

**Table 3: Calculation of the headcount index for the different functional forms**

| Name | Headcount index (c.d.f.) |
|---|---|
| Pareto | $P_0 = 1 - \left(\frac{z}{y_0}\right)^{-\alpha}$ |
| Lognormal | $P_0 = \Phi\left(\frac{z-\mu}{\sigma}\right)$ |
| Gamma | $P_0 = G(z, \rho, \gamma)$ |
| Weibull | $P_0 = 1 - e^{-\left(\frac{z}{\rho}\right)^{\beta}}$ |
| Fisk | $P_0 = \left(1 + \left(\frac{z}{\kappa}\right)^{-\tau}\right)^{-1}$ |
| Singh-Maddala | $P_0 = 1 - \left(1 + \left(\frac{z}{\kappa}\right)^{\tau}\right)^{\lambda}$ |
| Dagum | $P_0 = \left(1 + \left(\frac{z}{\kappa}\right)^{-\tau}\right)^{-\theta}$ |
| Beta 2 | $P_0 = B_2(z, \kappa, \lambda, \theta)$ |
| Kakwani and Podder (1973) | $(sP_0 + r){P_0}^{r} e^{-s(1-P_0)} = \frac{z}{\mu}$ |
| Chotikapanich (1993) | $P_0 = \frac{1}{k}\log\left(\frac{z\left(e^{k}-1\right)}{k\mu}\right)$ |
| Gaffney et al. (1980) | $\frac{\phi}{\zeta}\left(1-(1-P_0)^{\phi}\right)^{\frac{1}{\zeta}-1}(1-P_0)^{\phi-1} = \frac{z}{\mu}$ |
| Fernandez et al. (1991) | $\phi {P_0}^{\vartheta}(1-P_0)^{\phi-1} + \vartheta {P_0}^{\vartheta-1}\left(1-(1-P_0)^{\phi}\right) = \frac{z}{\mu}$ |
| Maddala and Singh (1977) | $-bd + a(1-b+bd){P_0}^{a-1} + bd(1-P_0)^{d-1} = \frac{z}{\mu}$ |
| Kakwani (1980) | $1 - \xi {P_0}^{\nu}(1-P_0)^{\upsilon}\left(\frac{\nu}{P_0} - \frac{\upsilon}{1-P_0}\right) = \frac{z}{\mu}$ |
| Arnold and Villaseñor (1989) | $P_0 = -\frac{1}{2m}\left(n + r\left(g + 2\frac{z}{\mu}\right)\right)\left(\sqrt{\left(g+2\frac{z}{\mu}\right)^2 - m}\right)^{-1}$ |
| | $w = -f - g - q - 1$ |
| | $m = g^2 - 4f$ |
| | $n = 2fw - 4q$ |
| | $r = \sqrt{n^2 - 4mw^2}$ |

Note: $\Phi$ stands for the c.d.f. of the standard normal distribution, $G$ for the c.d.f. of the Gamma distribution, $B_2$ for the c.d.f. of the Beta distribution of the second kind.

**Table 4: Mean value of growth and Gini elasticities of $P_0$, $P_1$ and $P_2$: whole sample.**

| Distribution | Growth elasticity | | | Gini elasticity | | | Valid |
|---|---|---|---|---|---|---|---|
| | $P_0$ | $P_1$ | $P_2$ | $P_0$ | $P_1$ | $P_2$ | estimations (%) |
| Pareto | 2 | 2.57 | 3.41 | -22.92 | -17.15 | -14.07 | 27 |
| | (5.29) | (9.63) | (10.52) | (11.24) | (9.82) | (14.15) | |
| Lognormal | -4.73 | -5.06 | -5.3 | 78.43 | 90.82 | 102.83 | 100 |
| | (0.14) | (0.13) | (0.15) | (2.97) | (3.93) | (3.8) | |
| Gamma | -1.81 | -1.89 | -1.94 | 24.53 | 34.65 | 44.7 | 100 |
| | (0.07) | (0.07) | (0.1) | (1.31) | (1.22) | (2.6) | |
| Weibull | -1.27 | -1.33 | -1.36 | 15.32 | 25.28 | 35.21 | 100 |
| | (0.05) | (0.05) | (0.05) | (0.64) | (0.92) | (1.03) | |
| Fisk | -2.21 | -2.33 | -2.41 | 27.09 | 37.09 | 47.04 | 100 |
| | (0.05) | (0.03) | (0.04) | (0.81) | (1.07) | (1.38) | |
| Beta 2 | -5.5 | -5.76 | -5.96 | 100.89 | 112.15 | 123.2 | 100 |
| | (0.86) | (0.87) | (26.54) | (24.76) | (21.1) | ($13 \times 10^2$) | |
| Singh-Maddala | -1.93 | -2.05 | -2.12 | 23.18 | 33.17 | 43.12 | 100 |
| | (0.07) | (0.08) | (0.67) | (1.11) | (1.37) | (9.19) | |
| Dagum | -1.88 | -2 | -2.08 | 22.43 | 32.42 | 42.37 | 100 |
| | (0.17) | (0.17) | (0.19) | (1.80) | (2.06) | (2.13) | |
| Kakwani and Podder (1973) | $-68 \times 10^6$ | -4.35 | -4.89 | $12 \times 10^7$ | 51.77 | 82.6 | 39 |
| | ($22 \times 10^9$) | (61.48) | (0.53) | ($2 \times 10^{11}$) | (39.73) | (4.06) | |
| Arnold and Villaseñor (1989) | -0.91 | -0.89 | -0.85 | 1.37 | 3.09 | 4.87 | 37 |
| | (22.24) | (57.75) | (16.5) | (300.73) | (720.72) | (176.07) | |
| Chotikapanich (1993) | 1.04 | 2.42 | 3.89 | -6.99 | -7.87 | -9.86 | 48 |
| | (5) | (16.11) | (15.87) | (19.01) | (73.36) | (83.06) | |
| Gaffney et al. (1980) | -2.09 | -2.09 | -2.17 | 23.73 | 33.48 | 43.45 | 94 |
| | (0.21) | (0.09) | (0.09) | (4.75) | (1.45) | (1.69) | |
| Fernandez et al. (1991) | -1.86 | -1.97 | -2.04 | 22.27 | 32.25 | 42.2 | 95 |
| | ($14 \times 10^4$) | (0.09) | (0.2) | ($7 \times 10^4$) | (1.44) | (1.75) | |
| Kakwani (1980) | -0.68 | -1.59 | -1.22 | 3.4 | 14.47 | 17.59 | 41 |
| | (0.13) | (2.28) | (0.46) | (0.99) | (17.57) | (3.05) | |
| Maddala and Singh (1977) | $40.3 \times 10^4$ | -2.6 | -2.61 | $-41 \times 10^5$ | 39.01 | 40.63 | 83 |
| | ($11 \times 10^6$) | (0.48) | (0.31) | ($14 \times 10^7$) | (4) | (1.98) | |

Note: bootstraped standard errors in parentheses.

Mean and median values of the elasticities are reported in table 5.

**Table 5: Mean value of growth and Gini elasticities of $P_0$, $P_1$ and $P_2$: valid common sample.**

| Distribution | Growth elasticity | | | Gini elasticity | | |
|---|---|---|---|---|---|---|
| | $P_0$ | $P_1$ | $P_2$ | $P_0$ | $P_1$ | $P_2$ |
| Lognormal | -4.09 | -4.42 | -4.67 | 56.12 | 67.31 | 78.14 |
| | (0.2) | (0.23) | (0.25) | (5.74) | (6.08) | (6.07) |
| Gamma | -1.66 | -1.75 | -1.81 | 18.33 | 27.49 | 36.57 |
| | (0.08) | (0.09) | (0.09) | (1.55) | (2.23) | (2.48) |
| Weibull | -1.24 | -1.3 | -1.34 | 12.52 | 21.52 | 30.5 |
| | (0.05) | (0.05) | (0.06) | (0.87) | (1.2) | (1.83) |
| Fisk | -2.16 | -2.29 | -2.36 | 22.58 | 31.62 | 40.62 |
| | (0.05) | (0.05) | (0.05) | (1.42) | (1.92) | (2.45) |
| Beta 2 | -3.12 | -3.39 | -3.58 | 34.24 | 44.05 | 53.66 |
| | (0.25) | (0.26) | (0.27) | (4.38) | (4.65) | (4.96) |
| Singh-Maddala | -1.85 | -1.98 | -2.05 | 18.07 | 27.12 | 36.12 |
| | (0.06) | (0.06) | (0.06) | (1.05) | (1.44) | (1.81) |
| Dagum | -1.75 | -1.88 | -1.96 | 16.33 | 25.37 | 34.38 |
| | (0.09) | (0.09) | (0.1) | (1.17) | (1.43) | (1.76) |
| Gaffney et al. (1980) | -2 | -2.01 | -2.09 | 18.29 | 27.09 | 36.11 |
| | (0.06) | (0.07) | (0.07) | (1.1) | (1.31) | (1.9) |
| Fernandez et al. (1991) | -1.77 | -1.89 | -1.96 | 17 | 26.03 | 35.03 |
| | (0.43) | (0.07) | (0.08) | (0.99) | (1.52) | (1.55) |
| Maddala and Singh (1977) | -2.25 | -2.46 | -2.61 | 21.5 | 30.9 | 40.17 |
| | (0.16) | (0.26) | (0.29) | (1.69) | (2.11) | (2.18) |

Note: bootstraped standard errors in parentheses. Common sample: 82.5% of initial observations; mean income: 6,963 $PPP; mean Gini: 0.39.

Differences between mean elasticities still remain important and frequently significant at the traditional level. In the case of the growth elasticity of the poverty headcount, values are ranged from $-4.09$ for the lognormal distribution to $-1.24$ for the Weibull distribution. However, as noted earlier, distributions generally exhibit different behaviors on the tails. As the absolute values of the elasticities of poverty increase rapidly with mean income, the differences stated in table 4 may only result from extreme values of the calculated elasticities. To "control" for these extreme values, we reported the median for each elasticity, functional form and poverty measure in table 6. Values are less heterogeneous, but we still notice significant differences.

In both tables 5 and 6, it appears that the lognormal distribution always provide the largest absolute mean and median values of both growth and Gini elasticities. On the contrary, lowest absolute values are provided by the Weibull distribution. However, even if most distributions lead to average growth elasticities close to 2 — a common value in the poverty-related literature — whatever poverty measure is considered, we cannot tell which value is

**Table 6: Median value of growth and Gini elasticities of $P_0$, $P_1$ and $P_2$: valid common sample.**

| Distribution | Growth elasticity | | | Gini elasticity | | |
|---|---|---|---|---|---|---|
| | $P_0$ | $P_1$ | $P_2$ | $P_0$ | $P_1$ | $P_2$ |
| Lognormal | -2.66 | -3.05 | -3.34 | 13.01 | 20.56 | 27.66 |
| | (0.2) | (0.2) | (0.21) | (1.3) | (1.66) | (1.97) |
| Gamma | -1.43 | -1.54 | -1.6 | 6.1 | 12.47 | 18.3 |
| | (0.12) | (0.13) | (0.12) | (0.69) | (0.87) | (1.04) |
| Weibull | -1.26 | -1.29 | -1.32 | 5.46 | 11.3 | 17.14 |
| | (0.08) | (0.08) | (0.08) | (0.44) | (0.73) | (0.97) |
| Fisk | -2.28 | -2.39 | -2.45 | 10.75 | 16.54 | 22.44 |
| | (0.08) | (0.09) | (0.08) | (0.75) | (0.98) | (1.26) |
| Beta 2 | -2.35 | -2.58 | -2.71 | 12.14 | 19.31 | 26.24 |
| | (0.11) | (0.11) | (0.12) | (1.28) | (1.6) | (1.9) |
| Singh-Maddala | -1.86 | -1.92 | -1.97 | 9.11 | 15.14 | 21.15 |
| | (0.06) | (0.07) | (0.06) | (0.69) | (1) | (1.37) |
| Dagum | -1.65 | -1.7 | -1.75 | 8.32 | 14.31 | 20.14 |
| | (0.09) | (0.09) | (0.1) | (0.67) | (0.97) | (1.21) |
| Gaffney et al. (1980) | -1.96 | -1.92 | -1.97 | 9.7 | 15.29 | 21.19 |
| | (0.07) | (0.07) | (0.08) | (0.63) | (0.97) | (1.39) |
| Fernandez et al. (1991) | -1.73 | -1.78 | -1.84 | 8.5 | 14.48 | 20.34 |
| | (0.4) | (0.08) | (0.08) | (0.69) | (0.99) | (1.24) |
| Maddala and Singh (1977) | -2.03 | -2.15 | -2.21 | 10.75 | 18.16 | 24.22 |
| | (0.21) | (0.17) | (0.21) | (1.29) | (1.55) | (2) |

Note: bootstraped standard errors in parentheses. Common sample: 82.5% of initial observations; mean income: 6,963 $PPP; mean Gini: 0.39.

the good one. Thus, we need to use some criterion to choose between the different functional forms the one that is the most appropriate for poverty analysis.

# 4 Which distribution should we choose?

As different distributions imply different results for the elasticities of poverty, the question is now to choose the distribution which corresponds best to the empirical distributions of our data set. We assume that we should converge to the true value of the elasticities as the quality of the fit improves. In the statistical size distribution literature, the traditional approach consists in using statistics based on the regression errors. In this way, we calculate the following traditional statistics of goodness-of-fit:

$$ssr = \sum_{i=1}^{N} \left(L(p_i) - \hat{L}(p_i)\right)^2, \tag{11}$$

$$sae = \sum_{i=1}^{N} \left|L(p_i) - \hat{L}(p_i)\right|, \tag{12}$$

with $\hat{L}$ standing for the estimated Lorenz curve.

A problem with these sums of squared ($ssr$) and absolute ($sae$) errors is that all errors are given the same weight. As Datt (1998) notes we are only interested in errors up to the value of the headcount index for the purpose of poverty analysis. In this way, he proposed using the following partial $ssr$:

$$pssr = \sum_{i=1}^{n} \left(L(p_i) - \hat{L}(p_i)\right)^2, \tag{13}$$

with $n$ corresponding to the first population quantile where $p_n \geqslant \hat{P}_0$. However, in the particular case of the headcount index, we are only interested in the quality of the fit in the vicinity of the estimated headcount[17]. Thus, we propose a measure based on squared error with weight decreasing with the distance from the estimated value of the headcount. To make comparisons feasible between each functional form, we normalize the measure by the sum of weights. It also allows us to do comparisons with the traditional $ssr$ statistic. This weighted $ssr$ is:

$$wssr = \frac{\sum_{i=1}^{N} \left(L(p_i) - \hat{L}(p_i)\right)^2 \left(1 - |p_i - \hat{P}_0|\right)^2}{\sum_{i=1}^{N} \left(1 - |p_i - \hat{P}_0|\right)^2}. \tag{14}$$

However, these criteria just focus on the precision of the estimation, but the use of more flexible functional forms may lead to an improvement of the fit that is not sufficient to compensate for the loss of degrees of freedom. In order to compare non-nested models while

[17] Remember that the headcount is defined as the point of the Lorenz curve where the slope is equal to the ratio of the poverty line to mean income, as in equation (9). So its elasticities depend only on the form of the Lorenz curve close to the estimated headcount.

penalizing for the addition of new parameters, we can use the Akaike and Schwartz information criteria (*cf.* (Gujarati 2004)). These measures are respectively:

$$aic = e^{2K/N} \frac{\sum_{i=1}^{N} \left(L(p_i) - \hat{L}(p_i)\right)^2}{N} \tag{15}$$

$$bic = N^{K/N} \frac{\sum_{i=1}^{N} \left(L(p_i) - \hat{L}(p_i)\right)^2}{N} \tag{16}$$

where $K$ is the number of estimated parameters.

To ease the comparison of the average value of these different statistics between each functional form, we computed the ratios of mean and median statistics for each distribution to the mean and median of the best-fitted distribution. Results are shown in table 7 for the whole sample (including non-valid estimations) and in table 8 for our restricted sample. It appears that all two-parameter distributions (Pareto, lognormal, gamma, Weibull, Fisk and Chotikapanich 1993) performs very poorly in comparison with three-parameter distributions (beta 2, Singh-Maddala, Dagum, Kakwani and Podder 1973, Gaffney et al. 1980 and Fernandez et al. 1991) and four-parameter distributions (Maddala and Singh 1977, Arnold and Villaseñor 1989 and Kakwani 1993). In particular, we can see that the lognormal is average about 5 to 10 times less precise than Kakwani (1980) and Maddala and Singh (1977). This result is not surprising since a single inequality parameter can hardly account for the observed heterogeneity of income distributions. The use of the Akaike and Schwartz information criteria is translated into a fall of the ratios of goodness of fit, but still justifies leaving two-parameter distributions in favor of more flexible functional forms.

Despite the obvious superiority of some functional forms, in particular the Kakwani (1980) and Maddala and Singh (1977) forms, we cannot reject the other distributions. If a distribution generally poorly fit the data, it does not mean that the fit is systematically poor. To get a more precise picture of the respective performance of each functional form, we ranked the different valid estimations for each observed distribution by their respective value for each statistic. The frequency[18] of first ranking and the median rank of each functional form are reported in table 9. We note that, even if two-parameter distributions are most of the time outperformed by more flexible functional forms, they sometime fit better than more flexible functional forms. In particular, the lognormal distribution is the best choice for 2.7% of the observed income distributions, according to our $wssr$ statistic. So lognormality is not the rule, but it can be the exception.

Before turning back to elasticities, we have to notice that the observed ranking inside each family of functional forms is rather surprising. An important number of studies have insisted on the merits of the different functional forms used in the present study but comprehensive comparisons of functional forms for income distributions and Lorenz curve are

[18]The counts of ranking for each functional form and each criterion are reported in tables 13 to 17. The number of observations in table 15 differs from those of the other tables since per capita income informations are missing for about 250 observations. Since calculation of the $wssr$ statistic requires this information, the numbers of observations for the ranks are lower.

**Table 7: Ratios of goodness-of-fit.**

| Distribution | *ssr* | | *sae* | | *wssr* | | *aic* | | *bic* | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Median | Mean | Median | Mean | Median | Mean | Median | Mean | Median |
| Pareto | 98.82 | 133.79 | 14.55 | 15.34 | 112.10 | 161.57 | 61.75 | 101.54 | 60.10 | 93.20 |
| Lognormal | 8.92 | 4.51 | 3.26 | 2.50 | 9.57 | 4.88 | 5.91 | 3.74 | 5.72 | 3.41 |
| Gamma | 30.17 | 20.16 | 6.53 | 5.24 | 33.92 | 21.86 | 18.71 | 17.28 | 18.23 | 15.57 |
| Weibull | 32.07 | 31.47 | 7.43 | 6.99 | 35.50 | 35.26 | 20.03 | 26.21 | 19.50 | 23.90 |
| Fisk | 9.50 | 5.03 | 3.37 | 2.56 | 9.10 | 4.87 | 6.31 | 3.88 | 6.10 | 3.60 |
| Beta 2 | 2.29 | 1.30 | 1.51 | 1.29 | 2.56 | 1.36 | 2.49 | 1.29 | 2.31 | 1.23 |
| Singh-Maddala | 1.88 | 1.38 | 1.40 | 1.30 | 1.97 | 1.56 | 1.63 | 1.18 | 1.59 | 1.13 |
| Dagum | 1.79 | 1.49 | 1.49 | 1.36 | 1.97 | 1.75 | 1.52 | 1.30 | 1.49 | 1.25 |
| Kakwani and Podder (1973) | 89.82 | 70.88 | 8.86 | 8.08 | 65.35 | 16.58 | 64.76 | 69.51 | 65.13 | 64.59 |
| Arnold and Villaseñor (1989) | 22032.78 | 1.40 | 12.92 | 1.28 | 113.35 | 1.31 | 49939.50 | 1.39 | 42958.82 | 1.38 |
| Chotikapanich (1993) | 48.11 | 34.53 | 7.90 | 6.46 | 53.90 | 42.35 | 29.45 | 28.88 | 28.73 | 26.08 |
| Gaffney et al. (1980) | 1.45 | 1.34 | 1.33 | 1.26 | 1.57 | 1.48 | 1.23 | 1.13 | 1.21 | 1.09 |
| Fernandez et al. (1991). | 1.64 | 1.52 | 1.47 | 1.40 | 1.84 | 1.87 | 1.38 | 1.29 | 1.35 | 1.24 |
| Kakwani (1980) | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Maddala and Singh (1977) | 1.07 | 1.05 | 1.06 | 1.07 | 1.06 | 1.04 | 1.12 | 1.05 | 1.11 | 1.04 |

**Table 8: Ratios of goodness-of-fit (restricted valid sample).**

| Distribution | *ssr* | | *sae* | | *wssr* | | *aic* | | *bic* | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Median | Mean | Median | Mean | Median | Mean | Median | Mean | Median |
| Lognormal | 8.49 | 4.41 | 3.15 | 2.37 | 10.21 | 5.01 | 5.74 | 3.81 | 5.54 | 3.51 |
| Gamma | 36.59 | 20.31 | 6.69 | 4.93 | 40.49 | 20.54 | 23.31 | 17.23 | 22.65 | 15.96 |
| Weibull | 38.94 | 30.63 | 7.61 | 6.55 | 43.40 | 34.05 | 25.03 | 26.35 | 24.29 | 24.43 |
| Fisk | 7.69 | 4.58 | 3.05 | 2.33 | 7.68 | 4.55 | 5.32 | 3.55 | 5.12 | 3.28 |
| Beta 2 | 1.77 | 1.21 | 1.36 | 1.17 | 2.22 | 1.28 | 1.83 | 1.20 | 1.72 | 1.14 |
| Singh-Maddala | 1.36 | 1.29 | 1.26 | 1.19 | 1.50 | 1.46 | 1.12 | 1.10 | 1.10 | 1.06 |
| Dagum | 1.60 | 1.38 | 1.37 | 1.25 | 1.85 | 1.63 | 1.30 | 1.17 | 1.28 | 1.14 |
| Gaffney et al. (1980) | 1.35 | 1.23 | 1.23 | 1.15 | 1.49 | 1.38 | 1.10 | 1.06 | 1.08 | 1.02 |
| Fernandez et al. (1991) | 1.52 | 1.40 | 1.37 | 1.27 | 1.72 | 1.72 | 1.22 | 1.21 | 1.20 | 1.17 |
| Maddala and Singh (1977) | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

**Table 9: Goodness of fit: frequency of first ranking and median rank.**

| **Distribution** | **Frequency of first ranking (%)** | | | | | **Median rank** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *ssr* | *sea* | *wssr* | *aic* | *bic* | *ssr* | *sea* | *wssr* | *aic* | *bic* |
| Pareto | 0.0 | 0.0 | 0.1 | 0.1 | 0.1 | 12 | 13 | 12 | 12 | 12 |
| Lognormal | 1.2 | 2.4 | 2.7 | 9.3 | 10.4 | 8 | 8 | 8 | 8 | 8 |
| Gamma | 0.9 | 0.7 | 1.0 | 5.0 | 5.3 | 9 | 9 | 9 | 9 | 9 |
| Weibull | 0.2 | 0.3 | 0.3 | 1.0 | 1.2 | 10 | 10 | 10 | 10 | 10 |
| Fisk | 0.0 | 1.0 | 0.3 | 4.6 | 5.3 | 8 | 8 | 8 | 8 | 8 |
| Beta 2 | 17.8 | 16.5 | 20.0 | 11.9 | 13.4 | 3 | 3 | 4 | 4 | 4 |
| Singh-Maddala | 8.9 | 8.5 | 6.5 | 13.0 | 12.6 | 4 | 4 | 5 | 3 | 3 |
| Dagum | 6.9 | 7.5 | 5.3 | 7.9 | 8.0 | 5 | 5 | 6 | 5 | 5 |
| Kakwani and Podder (1973) | 0.3 | 0.4 | 0.8 | 0.2 | 0.3 | 12 | 12 | 11 | 12 | 12 |
| Arnold and Villaseñor (1989) | 5.9 | 6.4 | 7.8 | 5.3 | 5.1 | 4 | 3 | 3 | 4 | 5 |
| Chotikapanich (1993) | 0.3 | 0.3 | 0.3 | 0.4 | 0.4 | 12 | 12 | 12 | 12 | 12 |
| Gaffney et al. (1980) | 3.5 | 5.5 | 5.4 | 10.4 | 9.8 | 4 | 4 | 4 | 4 | 4 |
| Fernandez et al. (1991) | 4.2 | 4.0 | 3.9 | 7.8 | 7.9 | 6 | 6 | 6 | 6 | 5 |
| Kakwani (1980) | 21.0 | 19.9 | 20.3 | 10.6 | 8.7 | 1 | 1 | 2 | 2 | 3 |
| Maddala and Singh (1977) | 29.0 | 26.7 | 25.5 | 12.5 | 11.6 | 2 | 2 | 2 | 4 | 4 |

scarce. Moreover, they rarely mix classical distributions and *ad hoc* Lorenz curves[19]. Using 82 distribution data sets at various years for 23 developed and middle-income countries, Bandourian et al. (2002) observed that the Weibull and Dagum were the best-fitting models for the two- and three-parameter distribution family, when opposed to the gamma, lognormal, generalized gamma, beta 1, beta 2 and Singh-Maddala distributions. Our results suggest that Fisk and lognormal distributions are the best two-parameter models and the Singh-Maddala and Gaffney et al. (1980), the best three-parameter models. For *ad hoc* Lorenz curves comparisons, Cheong (2002) compared the Kakwani and Podder (1976), Kakwani (1980), Gaffney et al. (1980), Fernandez et al. (1991) and Chotikapanich (1993) functional forms on US data from 1977 to 1983 and noticed that Gaffney et al. (1980) and Kakwani (1980) were the most powerful models. In the present study, we find that Kakwani (1980) is the functional form which generally fits best to our sample of distributions. However, for poverty analysis, we should prefer Maddala and Singh (1977) and Gaffney et al. (1980) forms since definition intervals are larger.

## 5 The lognormal case and the most plausible elasticities

In a recent paper, Lopez and Servèn (2006), using the Dollar and Kraay (2002) database, concluded that lognormality cannot be rejected for the estimation of income distributions. So, even if the lognormal distribution does not fit as well income distributions as other forms do, it may produce reasonable values for the elasticities of poverty. Nevertheless preceding results (*cf* tables 4 to 6) seem to contradict this assertion. To test this hypothesis, we propose a comparison between estimated lognormal elasticities and the values which seems the most plausible for each observation. To get these values, we simply keep the estimated elasticity corresponding to the best fitting[20] model for each observation. The composition of these series of elasticities is given by the first three columns of table 9. Thus, we get growth and Gini elasticities series for our different poverty measures from the $ssr$, $sae$ and $wssr$ statistics. Summary statistics for these series are given in table 10. Since we are only interested in the quality of the fit in the vicinity of the estimated headcount, the mixed-series based on the $wssr$ criterion will be our preferred series.

It appears that lognormal elasticities are most of the time higher in absolute value than those obtained in the mixed series. On average, the lognormal elasticities are about 1 to 2 percentage points higher in absolute value for the growth elasticities. Differences in Gini elasticities are really striking, in particular for the headcount index. Under the lognormality assumption, the average elasticity is 78.43, twice larger than our preferred value of the elasticity. Such a large difference cannot be attributed to extreme values since the same

[19] Most of the time, goodness-of-fit tests of *ad hoc* Lorenz curves include the lognormal distribution as a benchmark, but never more performing distributions

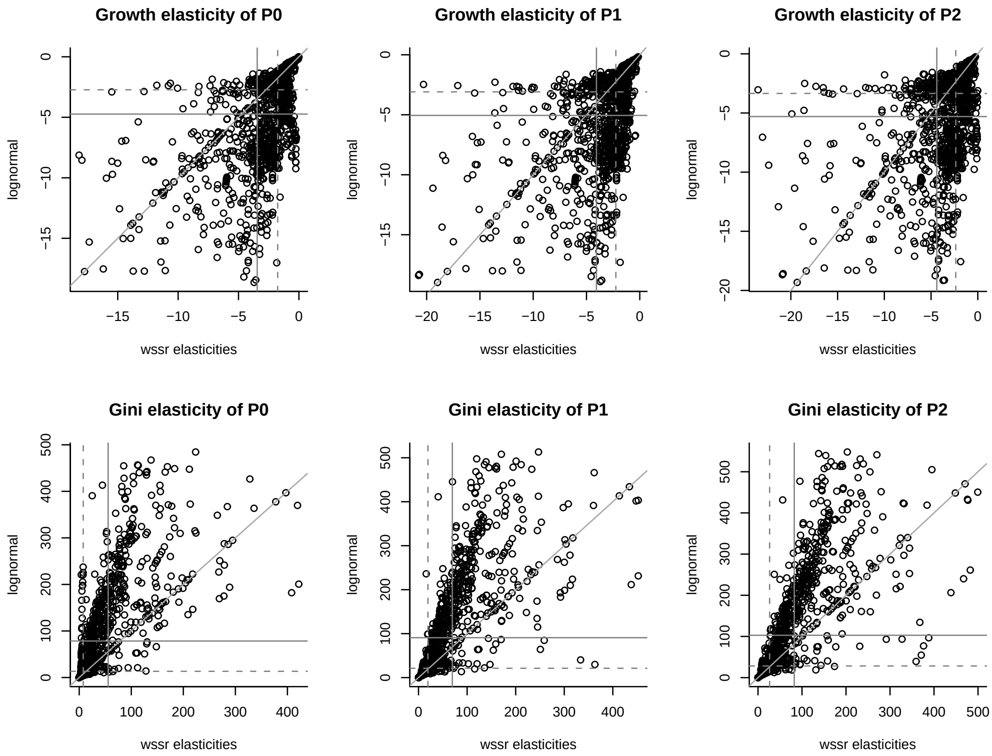
[20] As a robustness test, we also designed series corresponding to estimations that were ranked second according to our different statistics. Result for these second-best series do not differ from those obtained with best-fitting series.

**Table 10: Summary statistics for the mixed series based on the ssr, sae and wssr statistics.**

| Statistics | $P_0$ | | | $P_1$ | | | $P_2$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $ssr$ | $sea$ | $wssr$ | $ssr$ | $sea$ | $wssr$ | $ssr$ | $sea$ | $wssr$ |
| *Growth elasticities* | | | | | | | | | |
| Mean | −2.44 | −2.42 | −2.90 | −3.10 | −3.08 | −3.49 | −3.25 | −3.24 | −3.67 |
| Min | −34.11 | −34.11 | −55.33 | −51.03 | −51.03 | −55.72 | −49.35 | −49.35 | −56.09 |
| 1st quartile | −2.97 | −2.93 | −3.24 | −3.55 | −3.47 | −3.79 | −3.66 | −3.60 | −4.02 |
| Median | −1.54 | −1.58 | −1.74 | −2.13 | −2.12 | −2.22 | −2.20 | −2.17 | −2.34 |
| 3rd quartile | −0.89 | −0.90 | −0.91 | −1.26 | −1.26 | −1.29 | −1.26 | −1.26 | −1.30 |
| Max | −0.01 | −0.01 | −0.01 | −0.14 | −0.14 | −0.14 | 0.00 | 0.00 | 0.00 |
| Std deviation | 3.65 | 3.54 | 3.99 | 4.95 | 4.91 | 5.01 | 11.80 | 11.79 | 11.82 |
| *Gini elasticities* | | | | | | | | | |
| Mean | 33.49 | 32.73 | 41.69 | 46.92 | 46.10 | 54.96 | 56.90 | 56.24 | 65.26 |
| Min | −0.31 | −0.31 | −0.31 | 0.00 | 0.00 | 0.00 | 0.06 | 0.06 | 0.06 |
| 1st quartile | 1.30 | 1.30 | 1.30 | 4.03 | 4.03 | 3.99 | 6.52 | 6.49 | 6.52 |
| Median | 6.43 | 6.78 | 7.56 | 18.49 | 18.51 | 19.12 | 24.83 | 24.99 | 25.71 |
| 3rd quartile | 37.00 | 37.45 | 41.12 | 58.96 | 58.76 | 62.53 | 74.20 | 74.20 | 77.96 |
| Max | 1887.65 | 1887.65 | 1887.65 | 1957.24 | 1957.24 | 1957.24 | 2026.43 | 2026.43 | 2026.43 |
| Std deviation | 94.55 | 90.67 | 98.76 | 104.72 | 101.09 | 109.29 | 177.56 | 175.39 | 180.84 |

phenomenon is observed for median values. However, overestimation of the Gini elasticities tends to shrink as the $\theta$ parameter of the FGT measure increases.

Overestimation is a major problem, but we can imagine that lognormal elasticities are highly correlated with "true" elasticities, so that we can find the appropriate values of the desired elasticities through a simple linear correction. Figure 2 clearly show that the correlation between lognormal and the mixed series based on the $wssr$ criterion is rather low[21] (from 0.3 to 0.5 depending on the elasticity and the goodness-of-fit criterion). The plots also confirm that, on average, the lognormal assumption overestimates, in absolute value, the growth and Gini elasticities of poverty. So, we should be extremely cautious with simulations based on the lognormality assumption[22].




Note: Vertical and horizontal solid and dashed lines respectively correspond to mean and median values.

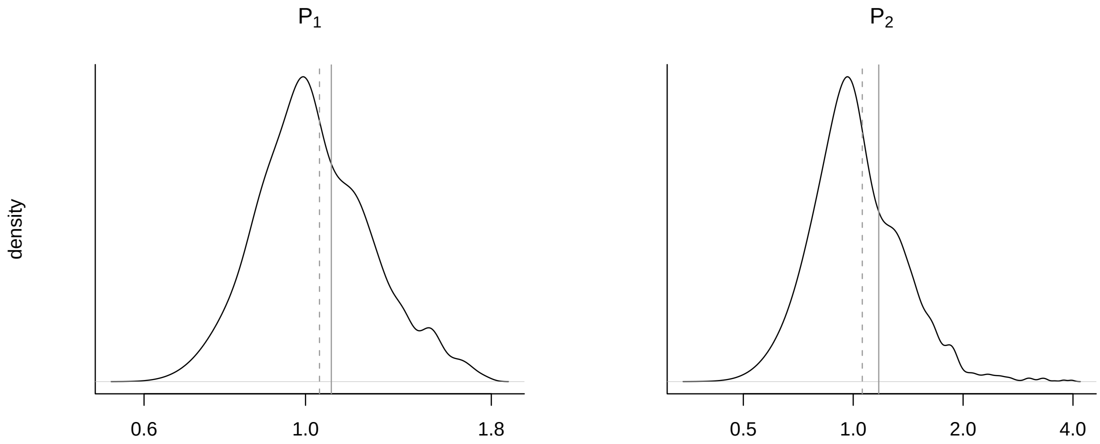
**Figure 2: Comparison of the estimated elasticities of poverty between lognormal distribution and wssr series.**

Finally, even if the lognormal distribution is not satisfying for a correct prediction of the magnitude of each elasticity, we may use it to compare the relative effectiveness of pure growth and pure distributional policies to achieve poverty alleviation. In section 2.1 (equation 7), we showed that under assumption (4) the ratio of the growth to Gini elasticities is

[21]The same differences are observed with the mixed series based on the $ssr$ and $sae$ statistics. Plots are reported on figures 6 and 7.

[22]Such risks are perfectly illustrated in CGE models by Boccanfuso et al. (2003)

independent of the distribution[23] when the headcount index is considered.This is not the case for higher values of the $\theta$ parameter of the FGT class of poverty measures. In order to check whether the lognormal assumption biases policy recommendations in favor of growth or distributional objectives, we can use the ratio of the lognormal elasticities ratio to the one corresponding to our best-fitting estimations. A value greater (lower) than unity for this policy bias ratio indicates that the lognormal assumption bias politics towards growth (inequality reduction). A kernel estimation of the density of this ratio for the $P_1$ and $P_2$ measures is reported on figure 3.




Note: Vertical solid and dashed lines respectively correspond to mean and median values.

**Figure 3: Gaussian kernel density of the policy bias ratio for $P_1$ and $P_2$ (lognormal against best-fitting estimations).**

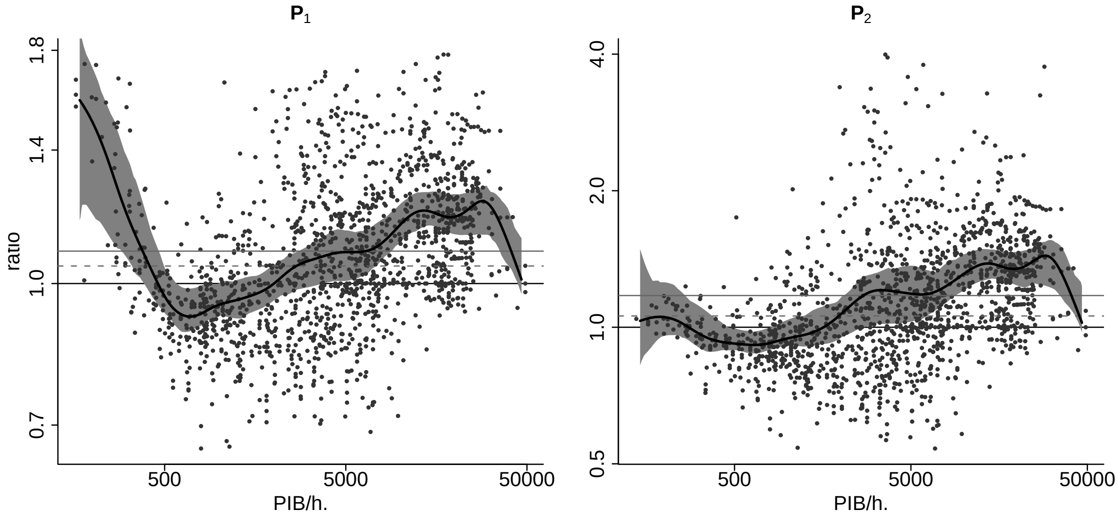
As the mean and median value of the policy bias ratio are close to unity, it seems that the use of the lognormality assumption does not systematically bias analysis in favor of a particular kind of policy. However, the variance of this ratio is large, in particular for the $P_2$ measure. Under the lognormal hypothesis, we notice that the relative importance of the growth elasticity can be overestimated or underestimated in excess of 50%. So we should be cautious with the use of the lognormal distribution to estimate the growth-inequality trade-off.

This large variance may be explained by the presence of high income countries in our sample. As poverty defined with a 2 $ PPP poverty line is essentially a concern for low income countries, it seems important to check if the mean value of our policy bias ratio and its variance vary with the level of development. On figure 4, we can observe the result of a non-parametric[24] estimation of the mean of this ratio conditional to income per capita. The gray area represents the 95% confidence interval of this conditional mean using a bootstrap

[23] Of course, this ratio is not independent of the distributional policy which can be used.

[24] Estimations are realized with a gaussian kernel. Bandwidth is chosen using the cross-validation procedure.

procedure. We notice that the estimated ratio is significantly different from unity for income greater than $4000. So, when the ratio of the per capita income on the poverty line is greater than 5, the use of the lognormal assumption bias our policy recommendations in favor the growth objective. Surprisingly we also observe an under-estimation of the ratio for values in the viciny of the poverty line.



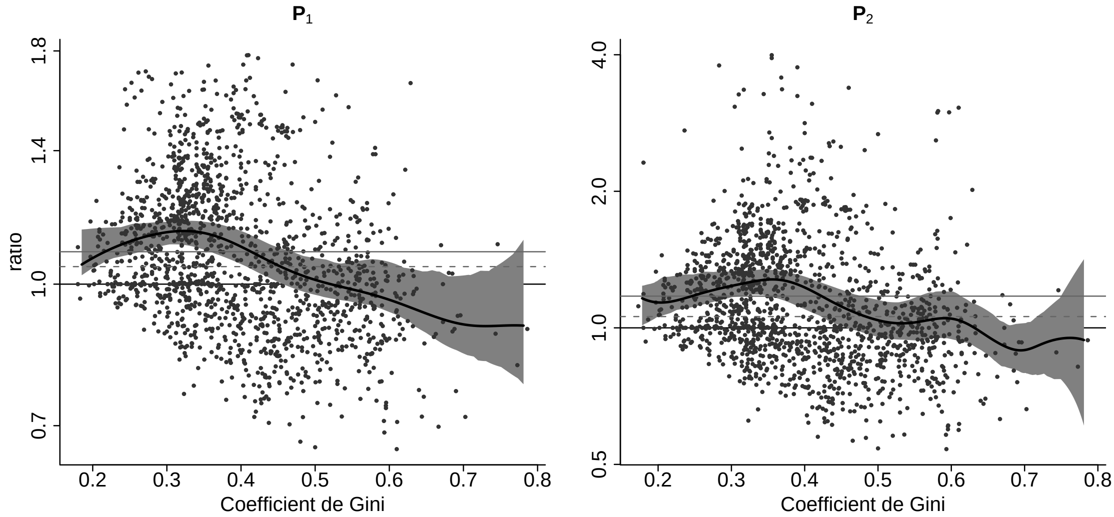

Note: solid and dashed lines respectively correspond to mean and median values. The thick line corresponds to the gaussian kernel estimation of the conditional mean of the political bias ratio. The gray area represents the 95% confidence interval of this conditional mean using a bootstrap procedure (500 replications, see note 16).

**Figure 4: GDP per capita versus policy bias ratio for $P_1$ and $P_2$ (lognormal against mixed-series wssr): non-parametric regression.**

Finally, to test if the lognormality assumption imply some policy bias depending on the degree of inequality, we also realized a non-parametric estimation of the mean of our policy bias ratio conditional to the inequality degree. Results are reported on figure 5. We can see that the relative effect of growth is over-estimated under the lognormality hypothesis when the Gini coefficient is less than 0.45. Since more than 70% of our sample includes distributions which exhibits Gini index that are less than 0.45, this result cannot be considered as trivial.

# 6 Concluding remarks

Throughout the present paper, we intended to answer the following questions. Which distributional hypothesis is suitable for a good estimation of the growth and inequality elasticities of poverty? What are the consequences of the use of a distributional assumption that does not suit to the observed heterogeneity of empirical income distributions? These are important questions since standard values of the elasticities of poverty are often used in applied studies like Collier and Dollar (2001), although uncertainty is great concerning the real val-




Note: solid and dashed lines respectively correspond to mean and median values. The thick line corresponds to the gaussian kernel estimation of the conditional mean of the political bias ratio. The gray area represents the 95% confidence interval of this conditional mean using a bootstrap procedure (500 replications, see note 16).

**Figure 5: Gini index versus policy bias ratio for $P_1$ and $P_2$ (lognormal against mixed-series wssr): non-parametric regression.**

ues. Attacking poverty in each developing country requires the design of policies that are appropriate to the income level and to the degree of inequality of each one of them. With no proper estimation of the trade-off between the intermediate objectives of growth and redistribution, we can doubt of the effectiveness of the policies that are recommended in order to reach the MDG.

For the first question, we conclude that in the absence of very flexible functional forms which parameters can easily be estimated, none of the tested distributions performs systematically better than the others for the description of observed income distributions, and so for poverty analysis. Pragmatism is required and we have to choose for each distribution the distributional assumption that fits best data. However, we can assert that two-parameter distributions should not be used in cross-section studies. More flexible functional forms are more appropriate for dealing with the heterogeneity of observed income distributions.

Our main result is that the use of poor distributional assumptions may induce some biases in the analysis of poverty. We chose to illustrate these biases with a comparison of the elasticities obtained through the popular lognormality assumption with those corresponding to the functional form that fits best each observed distribution. We notice that the use of the lognormal distribution leads to an overestimation of the effects of growth and inequality reduction in terms of poverty alleviation. Consequences are probably worse concerning the estimation of the relative size of these effects for the design of efficient policies aimed at poverty reduction. In particular we showed that resorting to the lognormal hypothesis introduces a significant bias in favor of growth oriented policies for high income or low and moderately unequal countries. Such biases may explain why it seems so difficult to reach the goals of poverty alleviation in many developing countries.

# Appendix

**Table 11: Distribution of the data in time and region.**

| Period | EAP | LAC | NA | MENA | SA | SSA | EEAC | WE | Total |
|---|---|---|---|---|---|---|---|---|---|
| 1950-54 | 4 | 7 | 0 | 1 | 11 | 0 | 0 | 4 | 27 |
| 1955-59 | 13 | 9 | 0 | 2 | 12 | 7 | 0 | 6 | 49 |
| 1960-64 | 23 | 29 | 0 | 3 | 15 | 8 | 5 | 21 | 104 |
| 1965-69 | 28 | 22 | 8 | 3 | 20 | 19 | 10 | 24 | 134 |
| 1970-74 | 35 | 36 | 11 | 2 | 17 | 1 | 9 | 39 | 150 |
| 1975-79 | 27 | 28 | 11 | 4 | 11 | 7 | 7 | 29 | 124 |
| 1980-84 | 32 | 27 | 10 | 1 | 9 | 7 | 8 | 46 | 140 |
| 1985-89 | 38 | 54 | 13 | 5 | 18 | 28 | 30 | 69 | 255 |
| 1990-94 | 49 | 70 | 13 | 6 | 6 | 58 | 43 | 74 | 319 |
| 1995-99 | 49 | 83 | 7 | 11 | 8 | 38 | 71 | 131 | 398 |
| 2000-02 | 11 | 20 | 5 | 3 | 3 | 7 | 12 | 81 | 142 |
| Total | 309 | 385 | 78 | 41 | 130 | 180 | 195 | 524 | 1842 |

EAP: East Asia and Pacific; LAC: Latin America and Caribbean; NA: North America; MENA: Middle East and North Africa; SA: South Asia; SSA: Sub-Saharan Africa; EECA: East Europe and Central Asia; WE: Western Europe.

**Table 12: Descriptive statistics of the dataset.**

| Statistic | GDP per capita ($PPP) | Gini coefficient |
|---|---|---|
| Mean | 7705,21 | 0,39 |
| Minimum | 138,77 | 0,17 |
| 1$^{st}$ quartile | 1667,04 | 0,32 |
| Median | 4673,94 | 0,37 |
| 3$^{rd}$ quartile | 11522,19 | 0,47 |
| Maximum | 48967,56 | 0,79 |

**Table 13: Goodness-of-fit rank: ssr statistic.**

| Distribution | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Pareto | 0 | 1 | 0 | 0 | 1 | 4 | 11 | 16 | 26 | 55 | 32 | 83 | 91 | 70 | 37 |
| Lognormal | 22 | 85 | 62 | 90 | 239 | 98 | 227 | 278 | 433 | 256 | 41 | 12 | 1 | 0 | 0 |
| Gamma | 16 | 39 | 35 | 38 | 58 | 254 | 117 | 153 | 302 | 324 | 450 | 58 | 0 | 0 | 0 |
| Weibull | 3 | 10 | 14 | 26 | 28 | 63 | 277 | 113 | 251 | 486 | 234 | 238 | 86 | 14 | 1 |
| Fisk | 0 | 1 | 34 | 129 | 123 | 133 | 241 | 442 | 296 | 233 | 128 | 55 | 28 | 1 | 0 |
| Beta 2 | 328 | 332 | 343 | 149 | 72 | 193 | 236 | 143 | 41 | 6 | 1 | 0 | 0 | 0 | 0 |
| Singh-Maddala | 165 | 257 | 352 | 384 | 310 | 243 | 101 | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Dagum | 127 | 208 | 202 | 285 | 302 | 287 | 262 | 116 | 51 | 4 | 0 | 0 | 0 | 0 | 0 |
| Kakwani and Podder (1973) | 6 | 4 | 0 | 0 | 0 | 0 | 5 | 2 | 13 | 38 | 165 | 121 | 131 | 134 | 3 |
| Arnold and Villaseñor (1989) | 108 | 123 | 67 | 94 | 73 | 53 | 36 | 21 | 12 | 6 | 1 | 2 | 0 | 0 | 0 |
| Chotikapanich (1993) | 5 | 3 | 0 | 4 | 2 | 1 | 2 | 3 | 10 | 33 | 153 | 325 | 225 | 2 | 0 |
| Gaffney et al. (1980) | 65 | 184 | 281 | 372 | 377 | 167 | 43 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Fernandez et al. (1991) | 77 | 126 | 147 | 155 | 190 | 313 | 260 | 175 | 58 | 1 | 0 | 0 | 0 | 0 | 0 |
| Kakwani (1980) | 387 | 95 | 79 | 48 | 26 | 11 | 4 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Maddala and Singh (1977) | 535 | 376 | 228 | 70 | 43 | 24 | 22 | 10 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |

**Table 14: Goodness-of-fit rank: sea statistic.**

| Distribution | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Pareto | 0 | 1 | 0 | 0 | 2 | 4 | 11 | 15 | 23 | 49 | 26 | 70 | 96 | 89 | 41 |
| Lognormal | 45 | 68 | 42 | 82 | 224 | 116 | 236 | 264 | 428 | 275 | 39 | 24 | 1 | 0 | 0 |
| Gamma | 12 | 29 | 32 | 36 | 53 | 267 | 135 | 145 | 307 | 332 | 449 | 45 | 2 | 0 | 0 |
| Weibull | 6 | 9 | 10 | 23 | 33 | 58 | 278 | 118 | 239 | 472 | 209 | 218 | 144 | 27 | 0 |
| Fisk | 19 | 31 | 63 | 147 | 129 | 144 | 223 | 384 | 280 | 189 | 123 | 72 | 34 | 6 | 0 |
| Beta 2 | 304 | 326 | 330 | 166 | 71 | 200 | 232 | 172 | 37 | 5 | 1 | 0 | 0 | 0 | 0 |
| Singh-Maddala | 156 | 256 | 360 | 350 | 316 | 247 | 120 | 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Dagum | 139 | 222 | 205 | 261 | 313 | 276 | 257 | 113 | 51 | 5 | 2 | 0 | 0 | 0 | 0 |
| Kakwani and Podder (1973) | 7 | 3 | 1 | 0 | 0 | 1 | 4 | 7 | 32 | 53 | 169 | 102 | 144 | 99 | 0 |
| Arnold and Villaseñor (1989) | 118 | 126 | 85 | 78 | 84 | 42 | 27 | 20 | 8 | 6 | 1 | 1 | 0 | 0 | 0 |
| Chotikapanich (1993) | 5 | 3 | 1 | 4 | 1 | 0 | 1 | 5 | 11 | 48 | 186 | 362 | 141 | 0 | 0 |
| Gaffney et al. (1980) | 101 | 172 | 253 | 369 | 354 | 177 | 59 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Fernandez et al. (1991) | 73 | 168 | 163 | 139 | 172 | 266 | 234 | 202 | 77 | 8 | 0 | 0 | 0 | 0 | 0 |
| Kakwani (1980) | 367 | 94 | 66 | 71 | 30 | 14 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Maddala and Singh (1977) | 492 | 336 | 233 | 118 | 62 | 32 | 17 | 18 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |

**Table 15: Goodness-of-fit rank: wssr statistic.**

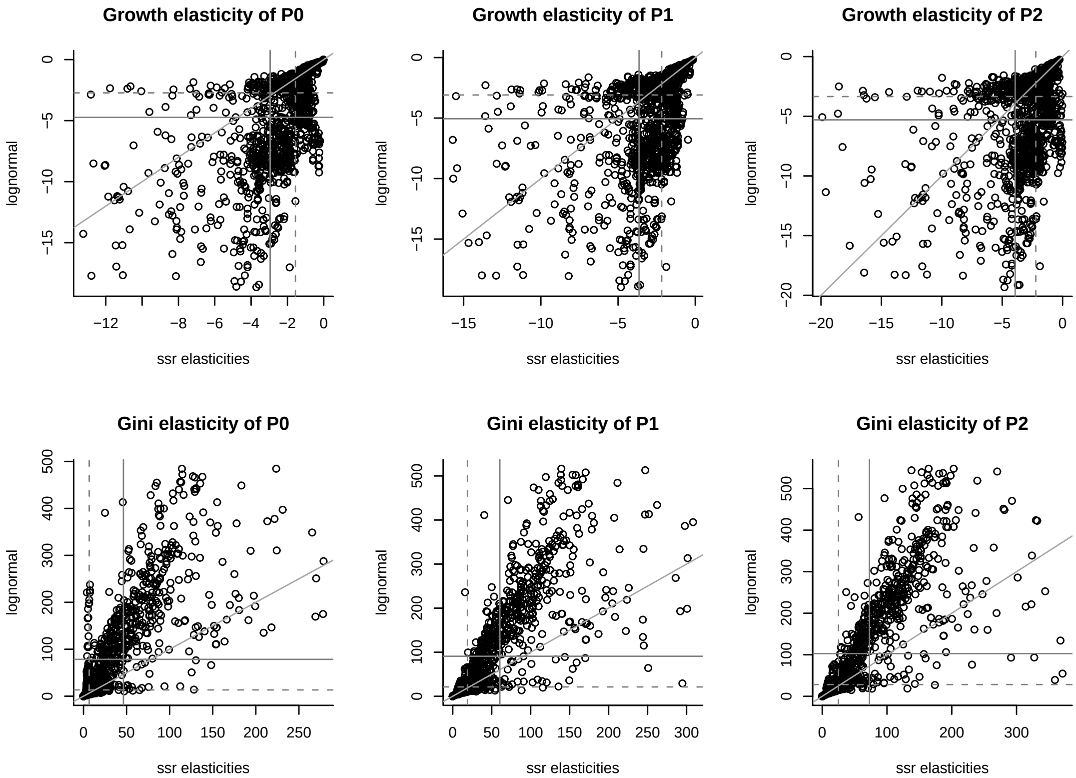
| Distribution | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Pareto | 1 | 0 | 3 | 2 | 1 | 1 | 15 | 15 | 32 | 60 | 50 | 80 | 69 | 63 | 35 |
| Lognormal | 42 | 82 | 81 | 55 | 108 | 69 | 188 | 219 | 347 | 285 | 73 | 23 | 6 | 0 | 0 |
| Gamma | 15 | 32 | 16 | 40 | 33 | 110 | 83 | 127 | 258 | 280 | 370 | 184 | 30 | 0 | 0 |
| Weibull | 4 | 9 | 14 | 13 | 16 | 17 | 92 | 86 | 225 | 386 | 265 | 208 | 182 | 61 | 0 |
| Fisk | 5 | 17 | 54 | 96 | 70 | 110 | 146 | 368 | 282 | 150 | 136 | 92 | 40 | 12 | 0 |
| Beta 2 | 316 | 295 | 224 | 120 | 66 | 135 | 195 | 162 | 53 | 12 | 0 | 0 | 0 | 0 | 0 |
| Singh-Maddala | 102 | 144 | 245 | 294 | 313 | 268 | 165 | 46 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Dagum | 83 | 134 | 130 | 153 | 258 | 265 | 282 | 172 | 79 | 16 | 6 | 0 | 0 | 0 | 0 |
| Kakwani and Podder (1973) | 12 | 7 | 2 | 2 | 5 | 6 | 14 | 46 | 88 | 124 | 85 | 80 | 77 | 68 | 6 |
| Arnold and Villaseñor (1989) | 123 | 116 | 63 | 98 | 69 | 49 | 33 | 20 | 10 | 7 | 6 | 2 | 0 | 0 | 0 |
| Chotikapanich (1993) | 5 | 3 | 0 | 4 | 1 | 6 | 0 | 5 | 20 | 110 | 214 | 225 | 158 | 17 | 0 |
| Gaffney et al. (1980) | 86 | 156 | 262 | 369 | 384 | 176 | 54 | 5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Fernandez et al. (1991) | 61 | 115 | 131 | 135 | 158 | 309 | 269 | 217 | 96 | 11 | 0 | 0 | 0 | 0 | 0 |
| Kakwani (1980) | 321 | 137 | 93 | 62 | 25 | 7 | 4 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Maddala and Singh (1977) | 402 | 331 | 260 | 135 | 71 | 50 | 38 | 19 | 4 | 1 | 0 | 0 | 0 | 0 | 0 |

**Table 16: Goodness-of-fit rank: aic statistic.**

| Distribution | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Pareto | 1 | 0 | 2 | 1 | 0 | 3 | 12 | 15 | 25 | 55 | 33 | 82 | 96 | 74 | 28 |
| Lognormal | 172 | 84 | 57 | 70 | 207 | 125 | 171 | 250 | 407 | 251 | 39 | 11 | 0 | 0 | 0 |
| Gamma | 92 | 28 | 31 | 34 | 53 | 258 | 102 | 132 | 283 | 323 | 450 | 58 | 0 | 0 | 0 |
| Weibull | 18 | 26 | 16 | 22 | 25 | 56 | 270 | 113 | 243 | 483 | 233 | 238 | 86 | 15 | 0 |
| Fisk | 85 | 46 | 83 | 121 | 128 | 121 | 186 | 369 | 266 | 245 | 119 | 47 | 27 | 1 | 0 |
| Beta 2 | 220 | 291 | 318 | 209 | 100 | 135 | 292 | 187 | 75 | 13 | 4 | 0 | 0 | 0 | 0 |
| Singh-Maddala | 239 | 315 | 373 | 296 | 267 | 194 | 117 | 42 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Dagum | 145 | 265 | 198 | 281 | 252 | 244 | 257 | 139 | 55 | 7 | 1 | 0 | 0 | 0 | 0 |
| Kakwani and Podder (1973) | 4 | 5 | 0 | 1 | 0 | 0 | 2 | 5 | 11 | 24 | 170 | 131 | 127 | 129 | 13 |
| Arnold and Villaseñor (1989) | 98 | 86 | 51 | 70 | 62 | 72 | 73 | 42 | 28 | 7 | 4 | 2 | 1 | 0 | 0 |
| Chotikapanich (1993) | 8 | 0 | 1 | 6 | 1 | 1 | 4 | 1 | 10 | 32 | 152 | 325 | 225 | 2 | 0 |
| Gaffney et al. (1980) | 191 | 271 | 246 | 310 | 309 | 128 | 35 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Fernandez et al. (1991) | 144 | 121 | 162 | 151 | 163 | 277 | 229 | 173 | 80 | 2 | 0 | 0 | 0 | 0 | 0 |
| Kakwani (1980) | 196 | 134 | 94 | 84 | 72 | 44 | 21 | 6 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Maddala and Singh (1977) | 231 | 172 | 212 | 188 | 205 | 186 | 73 | 33 | 11 | 0 | 0 | 0 | 0 | 0 | 0 |

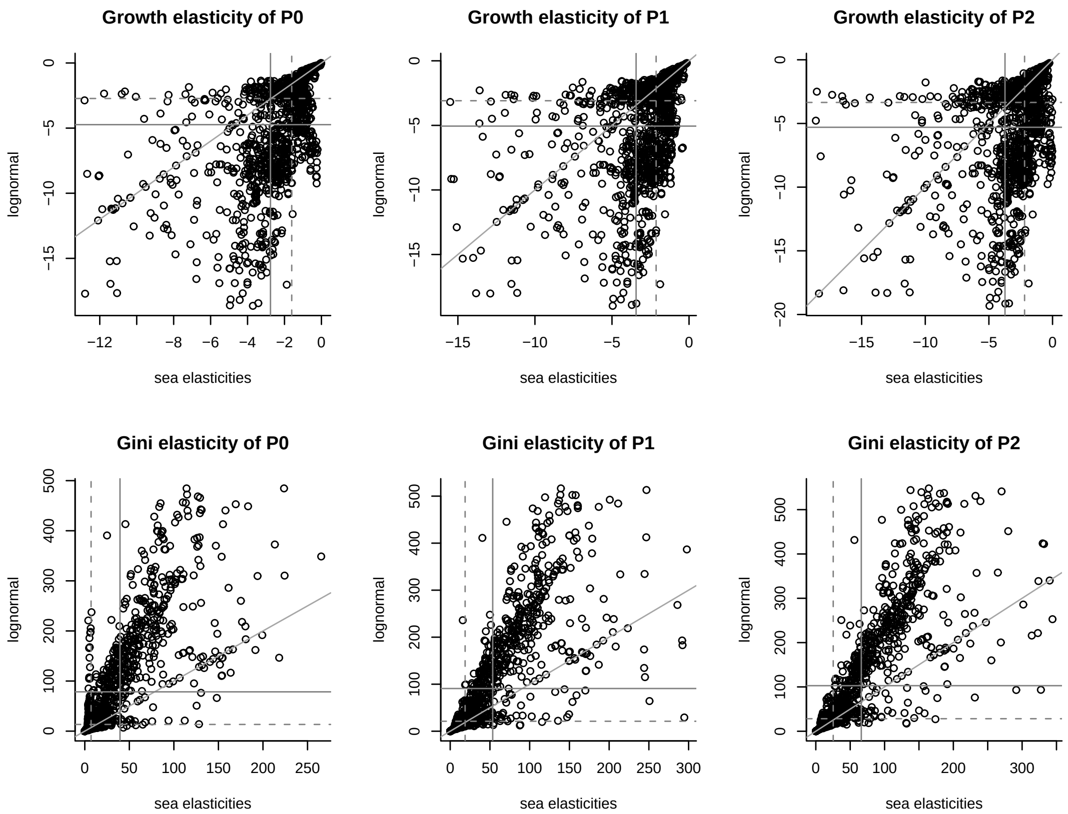
**Table 17: Goodness-of-fit rank: bic statistic.**

| Distribution | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Pareto | 1 | 0 | 3 | 0 | 0 | 3 | 11 | 16 | 25 | 55 | 33 | 82 | 97 | 74 | 27 |
| Lognormal | 191 | 74 | 58 | 69 | 209 | 121 | 169 | 248 | 408 | 249 | 37 | 11 | 0 | 0 | 0 |
| Gamma | 97 | 28 | 28 | 35 | 50 | 257 | 100 | 132 | 286 | 323 | 450 | 58 | 0 | 0 | 0 |
| Weibull | 22 | 22 | 16 | 24 | 23 | 57 | 272 | 108 | 245 | 483 | 233 | 238 | 86 | 15 | 0 |
| Fisk | 98 | 50 | 82 | 128 | 115 | 123 | 186 | 357 | 266 | 243 | 120 | 48 | 27 | 1 | 0 |
| Beta 2 | 247 | 320 | 318 | 189 | 92 | 127 | 292 | 182 | 60 | 13 | 4 | 0 | 0 | 0 | 0 |
| Singh-Maddala | 233 | 326 | 368 | 309 | 257 | 197 | 106 | 47 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Dagum | 148 | 263 | 204 | 294 | 239 | 229 | 260 | 140 | 59 | 6 | 2 | 0 | 0 | 0 | 0 |
| Kakwani and Podder (1973) | 6 | 3 | 0 | 1 | 0 | 0 | 4 | 3 | 11 | 26 | 169 | 130 | 126 | 129 | 14 |
| Arnold and Villaseñor (1989) | 94 | 76 | 51 | 68 | 57 | 85 | 63 | 52 | 35 | 7 | 5 | 2 | 1 | 0 | 0 |
| Chotikapanich (1993) | 7 | 1 | 1 | 6 | 0 | 2 | 4 | 2 | 9 | 32 | 152 | 325 | 225 | 2 | 0 |
| Gaffney et al. (1980) | 180 | 282 | 258 | 311 | 302 | 121 | 37 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Fernandez et al. (1991) | 145 | 121 | 169 | 155 | 171 | 266 | 218 | 171 | 81 | 5 | 0 | 0 | 0 | 0 | 0 |
| Kakwani (1980) | 161 | 124 | 95 | 80 | 111 | 46 | 27 | 7 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Maddala and Singh (1977) | 214 | 154 | 193 | 175 | 218 | 210 | 95 | 43 | 9 | 0 | 0 | 0 | 0 | 0 | 0 |




Note: Vertical and horizontal solid and dashed lines respectively correspond to mean and median values.

**Figure 6: Comparison of the estimated elasticities of poverty between lognormal distribution and ssr series.**




Note: Vertical and horizontal solid and dashed lines respectively correspond to mean and median values.

**Figure 7: Comparison of the estimated elasticities of poverty between lognormal distribution and sae series.**

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